Section 3.5: Using Newton's Laws

Tutorial 1 Practice, page 144

1. (a) Since the objects are not moving, the net force on each object is zero.
To calculate the tension F_{TA} in rope A, you can treat the two objects as one single object and ignore the tension F_{TB} in rope B since F_{TB} is an internal force in this case.

$$\vec{F}_{TA} = (m_A + m_B)\vec{g}$$
$$\vec{F}_{TA} = m_A\vec{g} + m_B\vec{g}$$

For rope B:

$$\vec{F}_{TB} = m_B\vec{g}$$

So, rope A has the greater tension.
(b) Let m_1 be the smallest mass, m_2 the middle mass, and m_3 the largest mass. The three masses are moving with the same acceleration a.
The only force acting on m_3 is the tension F_{TB} in rope B.

$$F_{TB} = m_3 a$$

Consider the net force acting on mass m_2.

$$\vec{F}_{TA} - \vec{F}_{TB} = m_2\vec{a}$$
$$\vec{F}_{TA} = m_2\vec{a} + \vec{F}_{TB}$$

So, rope A has the greater tension.
2. (a) The total mass m_T of the locomotive is

$$m_T = 6.4\times10^5\ \text{kg} + 5.0\times10^5\ \text{kg}$$
$$m_T = 1.14\times10^6\ \text{kg}$$

Choose east as positive. So, west is negative.

$$\vec{F}_{net} = m_T\vec{a}$$
$$F_{net} = (1.14\times10^6\ \text{kg})(-0.12\ \text{m/s}^2)$$
$$= -1.4\times10^5\ \text{N}$$
$$\vec{F}_{net} = 1.4\times10^5\ \text{N [W]}$$

The net force on the entire train is 1.4×10^5 N [W].
(b) The magnitude of the tension between the locomotive and the train car equals the magnitude of the net force on the train car.
The mass m_C of the train car is 5.0×10^5 kg.

$$F_{net} = m_C a$$
$$= (5.0\times10^5\ \text{kg})(0.12\ \text{m/s}^2)$$
$$F_{net} = 6.0\times10^4\ \text{N}$$

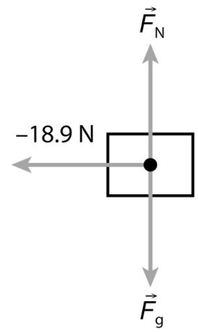
The magnitude of the tension between the locomotive and the train car is 6.0×10^4 N.

Tutorial 2 Practice, page 146

1. (a) Cart 1 and cart 2 are stuck together so they must move with the same acceleration.

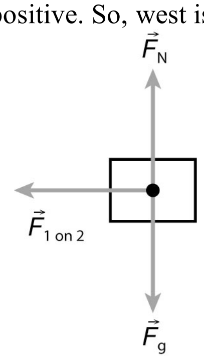
$$m_T = m_1 + m_2$$
$$= 1.2\ \text{kg} + 1.8\ \text{kg}$$
$$= 3.0\ \text{kg}$$

Assume no friction on the carts.
From the FBD of both boxes, the normal force and gravity cancel each other. Choose east as positive. So, west is negative.



$$F_{net} = m_T a$$
$$-18.9\ \text{N} = m_T a$$
$$-18.9\ \text{N} = (3.0\ \text{kg})a$$
$$a = \frac{-18.9\ \text{N}}{3.0\ \text{kg}}$$
$$= -6.3\ \text{m/s}^2$$
$$\vec{a} = 6.3\ \text{m/s}^2\ \text{[W]}$$

The acceleration of each cart is 6.3 m/s^2 [W].
(b) To calculate $F_{1\text{ on }2}$, draw the FBD for cart 2. Choose east as positive. So, west is negative.

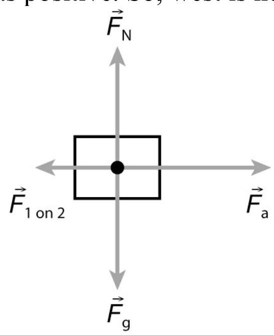


$$\vec{F}_{1\text{ on }2} = \vec{F}_{net}$$
$$\vec{F}_{1\text{ on }2} = m_2\vec{a}$$
$$F_{1\text{ on }2} = (1.8\ \text{kg})(-6.3\ \text{m/s}^2)$$
$$= -11\ \text{N}$$
$$\vec{F}_{1\text{ on }2} = 11\ \text{N [W]}$$

The force that cart 1 exerts on cart 2 is 11 N [W].
(c) If cart 2 were pushed with an equal but opposite force instead of cart 1, the net force on

the two carts would be 18.9 N [E]. The acceleration of each cart would be 6.3 m/s^2 [E].

To calculate $\vec{F}_{1\text{ on }2}$, draw the FBD for cart 2. Choose east as positive. So, west is negative.



$$\begin{aligned}\vec{F}_{\text{a}} + \vec{F}_{1\text{ on }2} &= \vec{F}_{\text{net}}\\ \vec{F}_{1\text{ on }2} &= \vec{F}_{\text{net}} - \vec{F}_{\text{a}}\\ F_{1\text{ on }2} &= m_2 a - (+18.9\text{ N})\\ &= (1.8\text{ kg})(+6.3\text{ m/s}^2) - 18.9\text{ N}\\ &= -7.6\text{ N}\\ \vec{F}_{1\text{ on }2} &= 7.6\text{ N [W]}\end{aligned}$$

The force that cart 1 exerts on cart 2 would be 7.6 N [W].

2. At first, the car moves at constant velocity for 0.50 s before the driver starts to slow down.

Given: $\vec{v}$ = 95 km/h [forward]; Δt = 0.50 s

Required: Δd_1

Analysis: Convert the velocity to SI units. Then use the equation $\Delta d_1 = \vec{v}\Delta t$ to determine the distance travelled. Choose forward as positive. So, backward is negative.

Solution:

$$v = +95\text{ km/h}$$
$$= \left(+95\,\frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1\,\cancel{\text{h}}}{60\,\cancel{\text{min}}}\right)\left(\frac{1\,\cancel{\text{min}}}{60\text{ s}}\right)\left(\frac{1000\text{ m}}{1\,\cancel{\text{km}}}\right)$$
$$v = +26.4\text{ m/s}$$
$$\vec{v} = 26.4\text{ m/s [forward] (one extra digit carried)}$$

$$\begin{aligned}\Delta d_1 &= v\Delta t\\ &= (+26.4\text{ m/s})(0.5\text{ s})\\ \Delta d_1 &= +13.2\text{ m (one extra digit carried)}\end{aligned}$$

Then, the car brakes with a net force of 2400 N [backward] for 2.0 s.

Given: m = 1200 kg; $\vec{F}_{\text{net}}$ = 2400 N [backward]; Δt = 2.0 s; $\vec{v}$ = 26.4 m/s [forward]

Required: Δd_2

Analysis: First calculate the acceleration of the car using $\vec{F}_{\text{net}} = m\vec{a}$. Use $\Delta d_2 = \vec{v}\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ to calculate the distance travelled. Choose forward as positive. So, backward is negative.

Solution:

$$\begin{aligned}F_{\text{net}} &= ma\\ -2400\text{ N} &= (1200\text{ kg})a\\ a &= \frac{-2400\text{ N}}{1200\text{ kg}}\\ a &= -2.0\text{ m/s}^2\end{aligned}$$

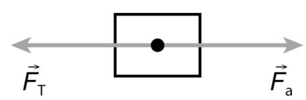
$$\begin{aligned}\Delta d_2 &= v\Delta t + \frac{1}{2}a\Delta t^2\\ &= (+26.4\text{ m/s})(2.0\text{ s}) + \frac{1}{2}(-2.0\text{ m/s}^2)(2.0\text{ s})^2\\ \Delta d_2 &= +48.8\text{ m (one extra digit carried)}\end{aligned}$$

Total distance travelled: 13.2 m + 48.8 m = 62 m

Statement: The total distance travelled by the car in 2.5 s is 62 m.

Section 3.5 Questions, page 147

1. (a) Draw a FBD of the rope.



The magnitude of the tension equals the magnitude of the applied force.

$$F_{\text{T}} = F_{\text{a}}$$
$$F_{\text{T}} = 65\text{ N}$$

The tension in the rope is 65 N.

(b) Use the same FBD of the rope in part (a). The tension in the rope is 65 N.

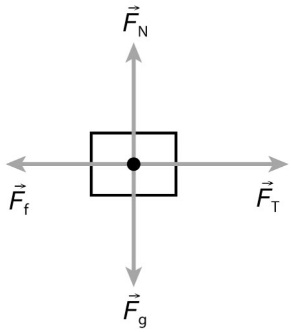
(c) Since there is no external force acting on the 12 kg object, the magnitude of the tension equals the magnitude of the applied force, which is 65 N.

2. (a) Given: m_1 = 72 kg; a = 2.0 m/s^2 [forward]; $\vec{F}_{\text{f}}$ = 120 N [backward]

Required: F_{T}

Analysis: Draw a FBD for the sled. Find F_{T} by adding all horizontal forces. Choose forward as positive. So, backward is negative.

Solution:



$$\begin{aligned} F_{net} &= F_T + F_f \\ m_1 a &= F_T + (-120\text{ N}) \\ (72\text{ kg})(+2.0\text{ m/s}^2) &= F_T - 120\text{ N} \\ F_T &= +260\text{ N} \end{aligned}$$

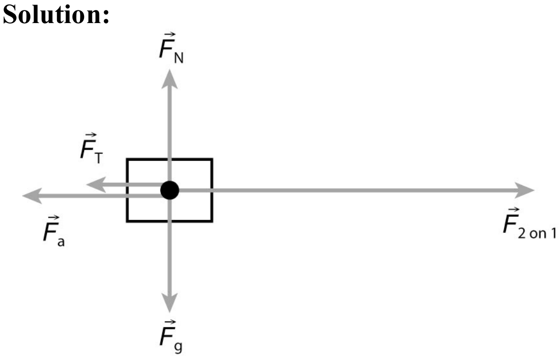
Statement: The tension in the rope is 260 N.

(b) Given: m_2 = 450 kg; a = 2.0 m/s^2 [forward]; $\vec{F}_T$ = 260 N [backward]; $\vec{F}_a$ = 540 N [backward]

Required: $\vec{F}_{2\text{ on }1}$

Analysis: Draw a FBD for the snowmobile. Find $\vec{F}_{2\text{ on }1}$ by adding all horizontal forces. Choose forward as positive. So, backward is negative.

Solution:



$$\begin{aligned} \vec{F}_{net} &= \vec{F}_{2\text{ on }1} + \vec{F}_a + \vec{F}_T \\ m_2 a &= F_{2\text{ on }1} + (-540\text{ N}) + (-260\text{ N}) \\ (450\text{ kg})(+2.0\text{ m/s}^2) &= F_{2\text{ on }1} - 540\text{ N} - 260\text{ N} \\ F_{2\text{ on }1} &= +1700\text{ N} \\ \vec{F}_{2\text{ on }1} &= 1700\text{ N [forward]} \end{aligned}$$

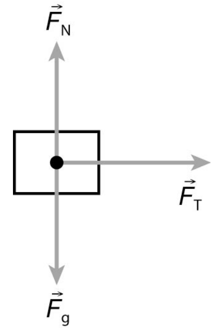
Statement: The force exerted by the snowmobile on the sled is 1700 N [forward].

3. Given: m = 70 kg; Δd = 15 m; $\vec{F}_T$ (rope on person) = 35 N [forward]

Required: Δt

Analysis: Draw a FBD for the skater reeling in the rope. First, find the acceleration of the person using $\vec{F}_{net} = m\vec{a}$. Use $\Delta d = \vec{v}\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ to calculate the time it takes the two skaters to meet. Choose forward as positive. So, backward is negative.

Solution:



$$\begin{aligned} F_{net} &= F_T \\ ma &= +35\text{ N} \\ (70\text{ kg})a &= +35\text{ N} \\ a &= +0.50\text{ m/s}^2 \\ \vec{a} &= 0.50\text{ m/s}^2\text{ [forward]} \end{aligned}$$

At the starting position, v = 0 m/s.

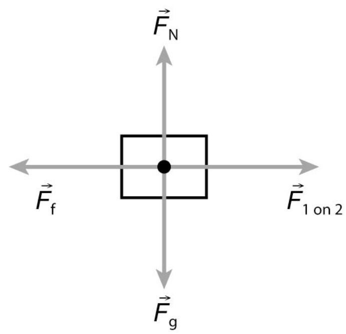
$$\begin{aligned} \Delta d &= v\Delta t + \frac{1}{2}a\Delta t^2 \\ \Delta d &= \frac{1}{2}a\Delta t^2 \\ \Delta t^2 &= \frac{2\Delta d}{a} \\ \Delta t &= \sqrt{\frac{2\Delta d}{a}} \\ &= \sqrt{\frac{2(15\ \cancel{\text{m}})}{0.50\ \cancel{\text{m}}/\text{s}^2}} \\ \Delta t &= 7.7\text{ s} \end{aligned}$$

Statement: It takes 7.7 s for the skaters to meet.

4. (a) Given: m_2 = 820 kg; $\vec{F}_f$ = 650 N [backward]; $\vec{F}_{net}$ = 0 N (at constant velocity)

Required: $\vec{F}_{1\text{ on }2}$

Analysis: Draw a FBD of the trailer with mass m_2. Find $\vec{F}_{1\text{ on }2}$ by adding all horizontal forces. Choose forward as positive. So, backward is negative.



Solution:

$\vec{F}_{1\text{ on }2} + \vec{F}_f = 0$

$\vec{F}_{1\text{ on }2} = -\vec{F}_f$

$F_{1\text{ on }2} = -(-650\text{ N})$

$F_{1\text{ on }2} = +650\text{ N}$

$\vec{F}_{1\text{ on }2} = 650\text{ N [forward]}$

Statement: The force that the car exerts on the trailer is 650 N [forward].

(b) Since the trailer is moving at constant velocity, the net force $\vec{F}_{net}$ on the trailer is 0 N.

The forces acting on the trailer are the same as in part (a). Therefore, the force that the car exerts on the trailer is 650 N [forward].

(c) Use the same FBD in part (a).
For the trailer accelerating at 1.5 m/s^2 [forward], $\vec{F}_{net} = m_2\vec{a}$. Find $\vec{F}_{1\text{ on }2}$ by adding all horizontal forces. Choose forward as positive. So, backward is negative.

$F_{1\text{ on }2} + F_f = F_{net}$

$F_{1\text{ on }2} + (-650\text{ N}) = m_2 a$

$F_{1\text{ on }2} - 650\text{ N} = (820\text{ kg})(+1.5\text{ m/s}^2)$

$F_{1\text{ on }2} = +1900\text{ N}$

$\vec{F}_{1\text{ on }2} = 1900\text{ N [forward]}$

The force that the car exerts on the trailer is 1900 N [forward].

(d) Use the same FBD in part (a).
For the trailer accelerating at 1.2 m/s^2 [backward], $F_{net} = m_2 a$. Find $F_{1\text{ on }2}$ by adding all horizontal forces. Choose forward as positive. So, backward is negative.

$F_{1\text{ on }2} + F_f = F_{net}$

$F_{1\text{ on }2} + (-650\text{ N}) = m_2 a$

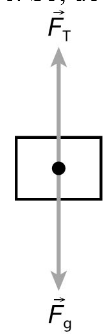
$F_{1\text{ on }2} - 650\text{ N} = (820\text{ kg})(-1.2\text{ m/s}^2)$

$F_{1\text{ on }2} = -330\text{ N}$

$\vec{F}_{1\text{ on }2} = 330\text{ N [backwards]}$

The force that the car exerts on the trailer is 330 N [backward].

5. Draw a FBD of the person climbing up the rope. Choose up as positive. So, down is negative.



Calculate the magnitude of the maximum tension in the rope.

$\vec{F}_T = m_r\vec{g}$

$F_T = (120\text{ kg})(9.8\text{ m/s}^2)$

$F_T = 1176\text{ N}$ (two extra digits carried)

From the FBD, the tension on the person is upward. Calculate the acceleration of the person.

$F_{net} = F_T + F_g$

$ma = +1176\text{ N} + mg$

$(85\text{ kg})a = +1176\text{ N} + (85\text{ kg})(-9.8\text{ m/s}^2)$

$a = +4.04\text{ m/s}^2$

$\vec{a} = 4.04\text{ m/s}^2\text{ [up]}$

Use $\Delta d = v\Delta t + \frac{1}{2}a\Delta t^2$ to calculate the time to climb the entire length of the rope.
At the starting position, $v = 0$ m/s.

$\Delta d = v\Delta t + \frac{1}{2}a\Delta t^2$

$\Delta d = \frac{1}{2}a\Delta t^2$

$\Delta t^2 = \frac{2\Delta d}{a}$

$\Delta t = \sqrt{\frac{2\Delta d}{a}}$

$= \sqrt{\frac{2(12.0\ \cancel{\text{m}})}{4.04\ \cancel{\text{m}}/\text{s}^2}}$

$\Delta t = 2.4\text{ s}$

The minimum time required to climb the entire length of the rope is 2.4 s.

6. Let $\vec{F}_{T1}$ = tension between force sensors 1 and 2, $\vec{F}_{T2}$ = tension between force sensors 3 and 4, and $\vec{F}_{T3}$ = tension between force sensors 5 and 6.

(a) Consider forces on cart 1 with mass 2.2 kg. Choose forward as positive. So, backward is negative.

$$\begin{aligned} F_{net} &= F_{T1} \\ m_1 a &= +3.3 \text{ N} \\ (2.2 \text{ kg})a &= +3.3 \text{ N} \\ a &= +1.5 \text{ m/s}^2 \\ \vec{a} &= 1.5 \text{ m/s}^2 \text{ [forward]} \end{aligned}$$

The acceleration of all the carts is 1.5 m/s^2 [forward].

(b) Use $\vec{F}_{T1}$ to calculate $\vec{F}_{T2}$. Consider forces on cart 2 with mass 2.5 kg. Choose forward as positive. So, backward is negative.

$$\begin{aligned} F_{net} &= F_{T1} + F_{T2} \\ m_2 a &= -3.3 \text{ N} + F_{T2} \\ (2.5 \text{ kg})(+1.5 \text{ m/s}^2) &= -3.3 \text{ N} + F_{T2} \\ F_{T2} &= +7.1 \text{ N} \\ \vec{F}_{T2} &= 7.1 \text{ N [forward]} \end{aligned}$$

Use $\vec{F}_{T2}$ to calculate $\vec{F}_{T3}$.

Consider forces on cart 3 with mass 1.8 kg.

$$\begin{aligned} F_{net} &= F_{T2} + F_{T3} \\ m_3 a &= -7.1 \text{ N} + F_{T3} \\ (1.8 \text{ kg})(+1.5 \text{ m/s}^2) &= -7.1 \text{ N} + F_{T3} \\ F_{T3} &= +9.8 \text{ N} \\ \vec{F}_{T3} &= 9.8 \text{ N [forward]} \end{aligned}$$

The reading on force sensor 1 is the same as the reading on force sensor 2, which is 3.3 N.
The reading on force sensor 2 is given as 3.3 N.
The reading on force sensor 3 is 7.1 N.
The reading on force sensor 4 is the same as the reading on force sensor 3, which is 7.1 N.
The reading on force sensor 5 is 9.8 N.
The reading on force sensor 6 is the same as the reading on force sensor 5, which is 9.8 N.

(c) The total mass m_T of the carts is:

$$m_T = 2.2 \text{ kg} + 2.5 \text{ kg} + 1.8 \text{ kg}$$
$$m_T = 6.5 \text{ kg}$$

Consider forces on force sensor 6. Choose forward as positive. So, backward is negative.

$$\begin{aligned} F_{net} &= F_a + F_{T3} \\ m_T a &= F_a + (-9.8 \text{ N}) \\ (6.5 \text{ kg})(+1.5 \text{ m/s}^2) &= F_a + (-9.8 \text{ N}) \\ F_a &= +20 \text{ N} \\ \vec{F}_a &= 20 \text{ N [forward]} \end{aligned}$$

The force applied to force sensor 6 is 20 N [forward].

7. The only force acting on car 2 is the tension F_{T2} between car 1 and car 2.

$$F_{T2} = m_2 a$$

F_{T1} is the tension between car 1 and the locomotive. Consider the net force acting on car 1.

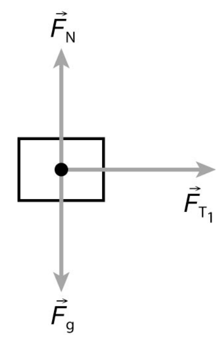
$$\begin{aligned} F_{T1} - F_{T2} &= m_1 a \\ F_{T1} &= m_1 a + F_{T2} \end{aligned}$$

So, F_{T1} is always greater than F_{T2}.

Given: $m_1 = 5.0 \times 10^5$ kg; $m_2 = 3.6 \times 10^5$ kg; $m = 6.4 \times 10^5$ kg; $F_{T1} = 2.0 \times 10^5$ N

Required: $\vec{a}$

Analysis: Since car 1 and car 2 are locked together, they can be treated as one single object. Draw a FBD of this object. Use the equation $\vec{F}_{net} = m_T\vec{a}$ to find the maximum acceleration. Choose forward as positive. So, backward is negative.



Solution: The total mass m_T of the two cars is:

$$\begin{aligned} m_T &= 5.0 \times 10^5 \text{ kg} + 3.6 \times 10^5 \text{ kg} \\ &= 8.6 \times 10^6 \text{ kg} \end{aligned}$$

$$\begin{aligned} F_{net} &= F_{T1} \\ m_T a &= +2.0 \times 10^5 \text{ N} \\ (8.6 \times 10^5 \text{ kg})a &= +2.0 \times 10^5 \text{ N} \\ a &= +0.23 \text{ m/s}^2 \\ \vec{a} &= 0.23 \text{ m/s}^2 \text{ [forward]} \end{aligned}$$

Statement: The maximum acceleration of the train that does not break the locking mechanism is 0.23 m/s^2 [forward].

8. (a) Given: m = 68 kg; F_{net} = 92 N; Δt_1 = 8.2 s

Required: $\vec{v}_1$

Analysis: First find the acceleration using the equation $\vec{F}_{net} = m\vec{a}$. Use $\vec{a} = \frac{\vec{v}_1 - \vec{v}_i}{\Delta t}$ to calculate $\vec{v}_1$.

Choose forward as positive. So, backward is negative.

Solution:

$$F_{net} = m_1 a_1$$
$$(68\text{ kg})a_1 = +92\text{ N}$$
$$a_1 = +1.35\text{ m/s}^2 \text{ (one extra digit carried)}$$

Since $v_i = 0$,

$$\vec{a}_1 = \frac{\vec{v}_1}{\Delta t_1}$$
$$v_1 = a_1\Delta t_1$$
$$= (+1.35\text{ m/s}^2)(8.2\text{ s})$$
$$= +11\text{ m/s}$$
$$\vec{v}_1 = 11\text{ m/s [forward]}$$

Statement: The speed of the skier is 11 m/s.

(b) Given: m = 68 kg; $\vec{v}_1$ = 11 m/s [forward]; Δt_2 = 3.5 s; $\vec{F}_{net}$ = 22 N [backward]

Required: $\vec{v}_2$

Analysis: First find the acceleration using the equation $\vec{F}_{net} = m\vec{a}$. Use $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ to calculate $\vec{v}_2$.

Choose forward as positive. So, backward is negative.

Solution:

$$F_{net} = m_2 a_2$$
$$(68\text{ kg})a_2 = -22\text{ N}$$
$$a_2 = -0.324\text{ m/s}^2 \text{ (one extra digit carried)}$$
$$a_2 = \frac{v_2 - v_1}{\Delta t_2}$$
$$v_2 - v_1 = a_2\Delta t_2$$
$$v_2 - (+11\text{ m/s}) = \left(-0.324\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(3.5\ \cancel{\text{s}})$$
$$v_2 = +10\text{ m/s}$$
$$\vec{v}_2 = 10\text{ m/s [forward]}$$

Statement: The speed of the skier is 10 m/s.

(c) Use $\Delta d = v\Delta t + \frac{1}{2}a\Delta t^2$ to calculate the distance travelled. Choose forward as positive. So, backward is negative.

For part (a), at the starting position, v = 0 m/s.

$$\Delta d_1 = \frac{1}{2}a_1\Delta t_1^2$$
$$= \frac{1}{2}(+1.35\text{ m/s}^2)(8.2\text{ s})^2$$
$$\Delta d_1 = +45.4\text{ m (one extra digit carried)}$$

For part (b), at the starting position, v = 11 m/s [forward].

$$\Delta d_2 = v\Delta t_2 + \frac{1}{2}a_2\Delta t_2^2$$
$$= \left(+11\ \frac{\text{m}}{\cancel{\text{s}}}\right)(3.5\ \cancel{\text{s}}) + \frac{1}{2}\left(-0.324\ \frac{\text{m}}{\cancel{\text{s}^2}}\right)(3.5\ \cancel{\text{s}})^{\cancel{2}}$$
$$\Delta d_2 = +36.5\text{ m (one extra digit carried)}$$

$$\Delta d = \Delta d_1 + \Delta d_2$$
$$= 45.4\text{ m} + 36.5\text{ m}$$
$$= 81.9\text{ m}$$
$$\Delta d = 82\text{ m}$$

Statement: The total distance travelled by the skier before coming to rest is 82 m.